

# The Mpemba Effect

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Aristotle, Bacon, Descartes, and in 1969, a Tanzania high school student named Mpemba experimentally observed that hot water can freeze faster than cold water. This effect is now called the Mpemba effect because Mpemba's experiments caused some controversy due to the non-intuitive nature of the effect. (See [http://math.ucr.edu/home/baez/physics/hot\\_water.html#Experiments](http://math.ucr.edu/home/baez/physics/hot_water.html#Experiments)) The Mpemba effect provides a marvelous concrete example of how important it is to test even the most common sense theories by experiment, and how difficult quantification and a complete characterization of even the simplest results can be.

Intuitively, many dismiss the Mpemba effect as impossible, because a hot water sample reaches the starting temperature of a cold water sample only after some delay, and thus the hot water sample must take longer to freeze. Amazingly, some individuals insist with almost religious zeal that it can not be so, without ever attempting an experiment or examining the literature. However, it is reasonable that the Mpemba effect is no illusion. There are sound reasons for the effect. A liquid is a dynamic system. Hot water carries more energy than cold water when it is placed into a freezer. Therefore, the convection eddy currents set up in the hot water are stronger than those set up in the cold water, and eddy currents in water last a very long time. Water is a fairly good insulator, so, all else being equal, the water with the most initial eddy currents at a given temperature is likely to freeze first - a hypothesis that is readily tested. The two samples involved in the Mpemba effect are not the same system when at the same temperature. One has greater kinetic energy than the other and therefore a greater cooling rate due to convection.

For example, a test sample consisting of completely still water (if that were possible) at 50 deg. C will be a different system when it reaches 10 deg. C than water which starts out as a control sample which is completely still water at 10 deg. C. In the process of the test sample temperature dropping to the starting temperature of the control sample, some of the heat energy of the test sample is converted to kinetic energy in the form of convection eddy currents.

Convection eddy currents are initiated spontaneously when there is a thermal gradient applied to a liquid. A thermal gradient converts some of the heat energy into the kinetic energy associated with the eddy currents. The eddy currents are therefore not induced by experimental error, but are necessitated by the mere fact that a thermal gradient must be induced in the water sample to cool it. To cool a sample the boundary of the sample must be made cooler than the interior, and thus

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a thermal gradient is necessarily formed by any experimental procedure that cools the water.

Once currents are established in water, they last a very long time, often for days, even though they become slow and normally undetectable. This can be shown experimentally by stirring water in a stoppered sink bowl in a specific direction and leaving it for a long period. When the stopper is removed a vortex will form as the water runs down the drain. The direction of the vortex will be the direction the water was stirred.

It may be possible that, by extreme effort, and by doing the experiment in a zero gravity situation, the effect might be nearly eliminated by nearly eliminating the convection eddy current formation due to the applied thermal gradient. However, some convection currents will always be initiated or amplified by the application of a thermal gradient to the sample. It is also extremely difficult to avoid momentarily applying a thermal gradient to the boundary itself, when the experiment is begun, and these gradients will be larger for a hot sample than for a cold sample, all else being equal.

Super cooling, the ability of water to drop below the freezing temperature when slowly cooled and very pure and still, should amplify the Mpemba effect. The more convection currents in a sample, the less super cooling should occur.

Without considerations of super cooling, water freezes at 0 deg. C, and requires about 79.8 calories per gram to make the transition from liquid to solid state, without any change in temperature involved. Liquid water requires only 1 calorie per gram to change 1 deg. C. It takes as much heat transfer to drop 79.8 deg. C as it does to freeze 0 deg. C water. If the starting temperature is below 79.8 deg. C, then the majority of the energy transfer occurs at 0 deg. C. The rate of heat transfer at 0 deg. C is a critical variable in determining which sample freezes first. Convection currents can dramatically affect heat transfer rates, by exposing large volumes of the liquid directly to the heat transfer boundary, be that the container walls or the ice itself. Slow moving molecules are culled out of the moving stream of water at the water-ice boundary. If the water does not move, then the relatively slower mechanism of thermal conduction is all that remains to effect the freezing.

If the heat transfer rate at 0 deg. C is only doubled by the increased convection, then

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water with an initial temperature of less than about 39.9 deg. C will freeze at about the same time as water initially at 0 deg. C. An almost 40 deg. advantage is given to the hotter water. That advantage is increased even further by the fact that the heat transfer rate depends on the difference between the temperature of the sample and the boundary temperature (the temperature of the freezer.) That difference is smallest when the sample is at 0 deg. C. The higher the freezer temperature, i.e. the closer to 0 deg. C, the greater the Mpemba effect should be.

It is difficult to cool a hot sample and a cold sample and have all else be equal. For example, without careful intervention, cold water has more dissolved gasses, and possibly more or less dissolved solids, some possibly from the container surface itself. Still, the effect occurs over a wide range of experimental conditions, and the reasons given above for the Mpemba effect remain in all experimental conditions.

There should not be much doubt that the observers of the effect, Mpemba, Aristotle, Bacon, and Descartes were correct, that at least under certain conditions hot water freezes faster than cold, all else being equal, but only experiment tells the tale for sure.